School:_____ Team:_____

**Directions:** (1) Provide units with all answers when appropriate.
(2) Give exact answers: do not round or approximate answers.
(3) Write student names and fully simplified answers on the lines provided.

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1. A diagonal of a square has endpoints $(-5, 3)$ and $(1, -4)$. Find the area of the square.

Student name: _____ Answer:_____

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2. A function $f$ is such that for each positive integer $n$,

$$f(n+1) = \frac{f(n)}{1 + af(n)},$$

where $a$ is a real number. Suppose $f(1) = 1$ and $f(11) = \frac{1}{2023}$. Find $a$.

Student name: _____ Answer:_____

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3. Find the surface area of the largest cube which fits inside a sphere of radius 1 inch.

Student name: _____ Answer:_____

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4. Find all positive integers $n$ for which $|x| + |x+1| + |x+2| + \cdots + |x+n| = n$ has an integer solution $x$.

Student name: _____ Answer:_____

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5. Simplify the following as much as possible:

$$\frac{1}{\log_2(2023!)} + \frac{1}{\log_3(2023!)} + \frac{1}{\log_4(2023!)} + \cdots + \frac{1}{\log_{2023}(2023!)}$$

Student name: _____ Answer:_____

School:_____ Team:_____

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1. Solve for $x$: $\log_{2023}(\log_4(\log_3 x)) = 0$

Student name: _____ Answer:_____

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2. What is the area of the region in the $xy$-plane described by the inequality $|x+y|+|x-y| \leq 2$?

Student name: _____ Answer:_____

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3. How many integers between 7,000 and 8,000 have the property that the digit in the thousands place value is the sum of the other three digits?

Student name: _____ Answer:_____

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4. Let $f(x) = kx^2 + kx + k$. Find all values of $k$ such that $f'(k) = 1$.

Student name: _____ Answer:_____

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5. Let $x$ be a real number such that $\sec(x) - \tan(x) = 2$. Determine the value of $\sec(x) + \tan(x)$.

Student name: _____ Answer:_____

School:_____ Team:_____

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1. Determine a formula for the quadratic function that passes through the points $(0, 3)$, $(1, -4)$, and $(2, -9)$.

Student name: _____ Answer:_____

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2. Evaluate $\int_0^3 \left(6 + \sqrt{9 - x^2}\right)\, dx$.

Student name: _____ Answer:_____

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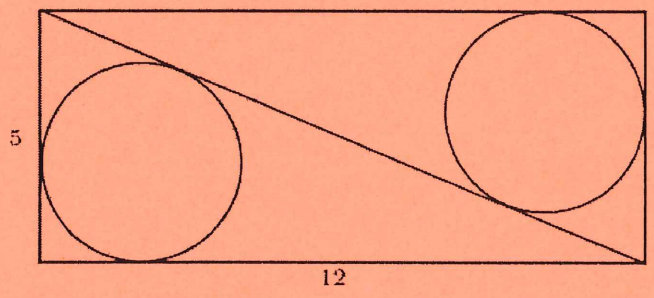
3. A $12 \times 5$ rectangle is divided into two right triangles by a diagonal. A circle is inscribed in each triangle. Find the distance between the centers of the two circles. No units are needed here.




Student name: _____ Answer:_____

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4. Evaluate $(1 + i)^5(1 - i)^5$.

Student name: _____ Answer:_____

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5. Find $f'\left(\frac{\pi}{6}\right)$ if $f(x) = \dfrac{\tan(x)}{1 - \tan^2(x)}$.

Student name: _____ Answer:_____

School:_____ Team:_____

**Directions:** (1) Provide units with all answers when appropriate.
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1. Find a positive number $k$ so that the line given by $y = k(x-2)$ intersects the parabola given by $y = x^2 - 5x + 10$ at exactly one point.

   Student name: _____ Answer:_____

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2. When Achilles started chasing a turtle, he was 990 yards behind the turtle. Achilles covers 10 yards in each second. The turtle covers 1 yard in 10 seconds. How long will it take Achilles to catch the turtle?

   Student name: _____ Answer:_____

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3. The radiator in Calista's truck contains 6.3 quarts of a mixture of antifreeze and water. The mixture is 30% antifreeze. How much of this mixture should Calista drain from her radiator and replace with pure antifreeze so that the final mixture will be 50% antifreeze?

   Student name: _____ Answer:_____

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4. Find the value of $y$ such that the line passing through the points $(-4, y)$ and $(11, 12)$ is perpendicular to the line containing $(1, 5)$ and $(4, -3)$.

   Student name: _____ Answer:_____

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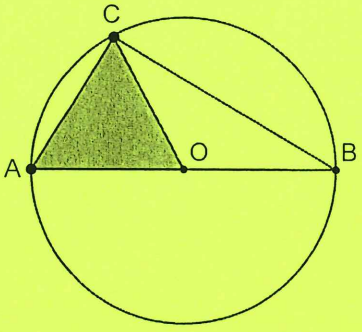
5. Consider a circle with center $O$ and triangle $\Delta ABC$ as shown below (figure not drawn to scale). Suppose $OA = 5\sqrt{2}$ cm and $BC = 14$ cm. What is the area of the shaded region?




   Student name: _____ Answer:_____

School:_____ Team:_____

**Directions:** (1) Provide units with all answers when appropriate.
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1. Simplify as far as possible: $\dfrac{17!}{\sqrt{15!\,16!}}$

Student name: _____ Answer:_____

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2. Evaluate $\lim\limits_{n\to\infty} \dfrac{1+2+3+\cdots+n}{3n^2-n+19}$

Student name: _____ Answer:_____

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3. Solve $1+\dfrac{x-1}{x-3} > \dfrac{x-2}{x-1}$. Write your answer using interval notation.

Student name: _____ Answer:_____

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4. Evaluate: $\sin^4(15°) - \cos^4(15°)$

Student name: _____ Answer:_____

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5. Find the height of a regular tetrahedron with sides of length 1 inch.

Student name: _____ Answer:_____